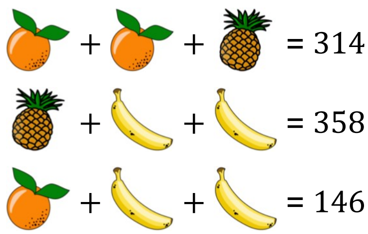
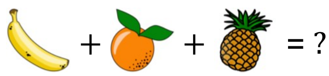
Exam 3

1. **Question**

   Given the following information:

   Compute:

   a. 314
   b. 246
   c. 336
   d. 262
   e. 316

   Solution

   The information provided can be interpreted as the price for three fruit baskets with different combinations of the three fruits. This corresponds to a system of linear equations where the price of the three fruits is the vector of unknowns x:

   $x_1 =$  $x_2 =$  $x_3 =$ 

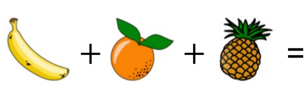
   The system of linear equations is then:

   $$\begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 314 \\ 358 \\ 146 \end{pmatrix}$$

   This can be solved using any solution algorithm, e.g., elimination:

   $$x_1 = 56,\ x_2 = 34,\ x_3 = 246.$$

   Based on the three prices for the different fruits it is straightforward to compute the total price of the fourth fruit basket via:

   $x_1 + x_2 + x_3 =$

$56 + 34 + 246 = 336$

a. False
b. False
c. True
d. False
e. False

2. **Question**

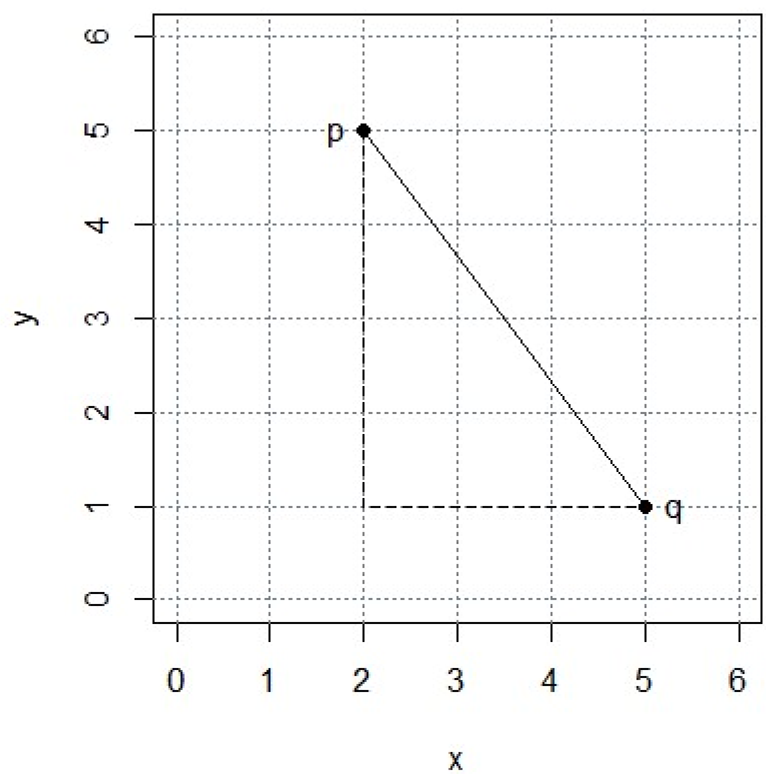
What is the distance between the two points $p = (2, 5)$ and $q = (5, 1)$ in a Cartesian coordinate system?

a. 6.322
b. 8.865
c. 7.822
d. 2.646
e. 5.000

Solution

The distance d of p and q is given by $d^2 = (p_1 - q_1)^2 + (p_2 - q_2)^2$ (Pythagorean formula).

Hence $d = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2} = \sqrt{(2-5)^2 + (5-1)^2} = 5.$



a. False
b. False
c. False
d. False
e. True

3. **Question**

What is the derivative of $f(x) = x^3 e^{3.3x}$, evaluated at $x = 0.57$?

Solution

Using the product rule for $f(x) = g(x) \cdot h(x)$, where $g(x) := x^3$ and $h(x) := e^{3.3x}$, we obtain

$$\begin{aligned} f'(x) &= [g(x) \cdot h(x)]' = g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ &= 3x^{3-1} \cdot e^{3.3x} + x^3 \cdot e^{3.3x} \cdot 3.3 \\ &= e^{3.3x} \cdot (3x^2 + 3.3x^3) \\ &= e^{3.3x} \cdot x^2 \cdot (3 + 3.3x). \end{aligned}$$

Evaluated at $x = 0.57$, the answer is

$$e^{3.3 \cdot 0.57} \cdot 0.57^2 \cdot (3 + 3.3 \cdot 0.57) = 10.403188.$$

Thus, rounded to two digits we have $f'(0.57) = 10.40$.

4. **Question**

The daily expenses of summer tourists in Vienna are analyzed. A survey with 133 tourists is conducted. This shows that the tourists spend on average 196.7 EUR. The sample variance s_{n-1}^2 is equal to 136.7.

Determine a 95% confidence interval for the average daily expenses (in EUR) of a tourist.

a. What is the lower confidence bound?
b. What is the upper confidence bound?

Solution

The 95% confidence interval for the average expenses μ is given by:

$$\begin{aligned} & \left[\bar{y} - 1.96\sqrt{\frac{s_{n-1}^2}{n}},\ \bar{y} + 1.96\sqrt{\frac{s_{n-1}^2}{n}}\right] \\ = & \left[196.7 - 1.96\sqrt{\frac{136.7}{133}},\ 196.7 + 1.96\sqrt{\frac{136.7}{133}}\right] \\ = & [194.713,\ 198.687]. \end{aligned}$$

a. The lower confidence bound is 194.713.
b. The upper confidence bound is 198.687.

5. **Question**

For 53 firms the number of employees X and the amount of expenses for continuing education Y (in EUR) were recorded. The statistical summary of the data set is given by:

	Variable X	**Variable** Y
Mean	45	266
Variance	120	2231

The correlation between X and Y is equal to 0.73.

Estimate the expected amount of money spent for continuing education by a firm with 45 employees using least squares regression.

Solution

First, the regression line $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ is determined. The regression coefficients are given by:

$$\hat{\beta}_1 = r \cdot \frac{s_y}{s_x} = 0.73 \cdot \sqrt{\frac{2231}{120}} = 3.14762,$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \cdot \overline{x} = 266 - 3.14762 \cdot 45 = 124.35719.$$

The estimated amount of money spent by a firm with 45 employees is then given by:

$$\hat{y} = 124.35719 + 3.14762 \cdot 45 = 266.$$